

Section 5.1

Definition of the Natural Logarithmic Function: The natural logarithmic function is defined by

$$\ln x = \int_1^x \frac{1}{t} dt, \quad x > 0.$$

The domain of the natural logarithmic function is the set of all positive real numbers.

Properties of the Natural Logarithmic Function: The natural logarithmic function has the following properties:

1. The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.
2. The function is continuous, increasing, and one-to-one.
3. The function is concave downward.

Logarithmic Properties: If a and b are positive numbers and n is rational, then the following properties are true:

1. $\ln(1) = 0$
2. $\ln(ab) = \ln a + \ln b$
3. $\ln(a^n) = n \ln a$
4. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

Definition of e : The letter e denotes the positive real number such that

$$\ln e = \int_1^e \frac{1}{t} dt = 1.$$

Derivative of the Natural Logarithmic Function: Let u be a differentiable function of x .

1. $\frac{d}{dx} [\ln x] = \frac{1}{x}, \quad x > 0$
2. $\frac{d}{dx} [\ln u] = \frac{1}{u} \frac{du}{dx} = \frac{u'}{u}, \quad u > 0$

1) Expand the following logarithmic expressions.

a) $\ln \sqrt[3]{2x-1}$

b) $\ln\left(\frac{17x}{x-2}\right)$

c) $\ln \frac{\sqrt{x^2+3}}{2(x-2)^3}$

2) Find the following:

a) $\frac{d}{dx} [\ln(3x^2)]$

b) $\frac{d}{dx} [\ln(\sin x)]$

c) $\frac{d}{dx} [(\ln(2x - 1))^2]$

d) $\frac{d}{dx} [\ln \sqrt[3]{3x + 4}]$

3) Differentiate $f(x) = \ln \left[\frac{2\sqrt{3x^2+1}}{x^2(x+1)^3} \right]$.

4) Use logarithmic differentiation to find the derivative of $g(x) = \frac{(x^2-4)^3}{\sqrt[3]{x^4-2}}$

5) Find the derivative of $f(x) = \ln|\tan x|$. Simplify the derivative using trigonometric identities.